NOTES SEQUENCES AND SERIES:

1. Quadratic number pattern: (Second difference is a constant)

- General Term: $T_n = an^2 + bn + c$
- There is NO formulae for the Sum of a Quadratic number Pattern:
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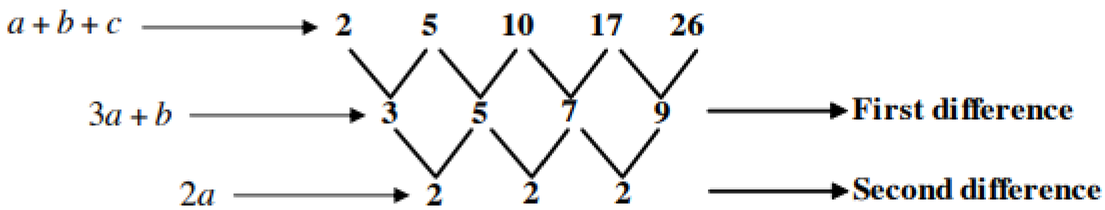
So, consider the previous number pattern: 2; 5; 10; 17; 26; ……..



It is clearly a quadratic number pattern because it has a constant second difference. You can now proceed as follows:

$2a = 2$	$3a + b = 3$	$a + b + c = 2$
$\therefore a = 1$	$\therefore 3(1) + b = 3$	$\therefore 1 + 0 + c = 2$
	$\therefore b = 0$	$\therefore c = 1$

Therefore the general term is $T_n = n^2 + 0n + 1 = n^2 + 1$

EXAMPLE:

Consider the following number pattern: 6; 17; 34; 57; ……………………

(a) Show that it is a quadratic number pattern.

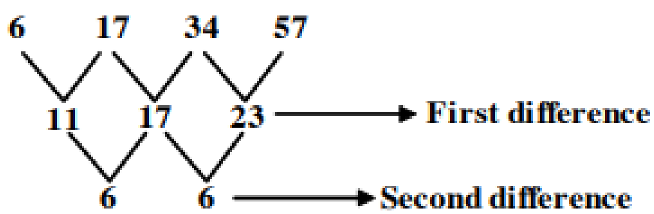
(b) Write down the next two terms of the number pattern.

(c) Hence determine the nth term as well as the 100th term.

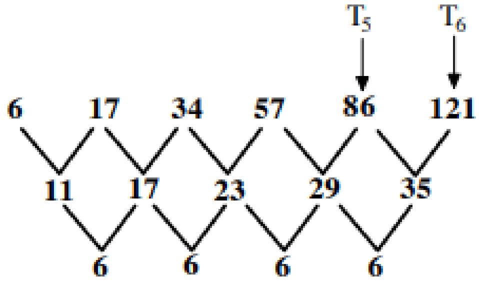
(d) Determine which term equals 162.

Solution

(a)



(b) The fourth term in the second row can be determined by adding 6 to 23 to get 29. It will then be possible to determine the fifth term of the number pattern by adding 29 to 57 which gives 86. This process is repeated to give the sixth term of the number pattern.



(c) $2a$ = second difference

$\therefore 2a = 6$ $\quad 3a + b = 11$ $\quad a + b + c = 6$

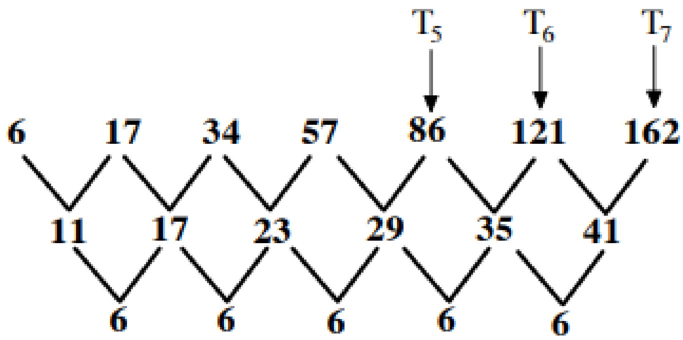
$\therefore a = 3$ $\quad \therefore 3(3) + b = 11$ $\quad \therefore 3 + 2 + c = 6$

$\therefore b = 2$ $\quad \therefore c = 1$

$\therefore T_n = 3n^2 + 2n + 1$

$\therefore T_{100} = 3(100)^2 + 2(100) + 1 = 30201$

(d) You could have continued the process done in (b) to find out which term equals 162. Clearly, the 7^{th} term is equal to 162.



Alternatively, you could have used quadratic equations to assist you:

$T_n = 162$

$\therefore 3n^2 + 2n + 1 = 162$

$\therefore 3n^2 + 2n - 161 = 0$

$\therefore (3n + 23)(n - 7) = 0$

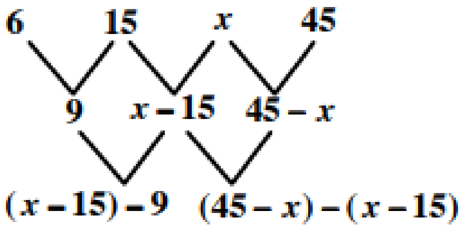
$\therefore n = -\frac{23}{3}$ or $n = 7$

But $n \neq -\frac{23}{3}$

$\therefore n = 7$

EXAMPLE

$6 ; 15 ; x ; 45 ; ....$ is a quadratic number pattern (sequence). Determine the value of x.



$$(x-15)-9=(45-x)-(x-15)$$
$$\therefore x-24=45-x-x-15$$
$$\therefore x-24=60-2x$$
$$\therefore 3x=84$$
$$\therefore x=28$$

EXAMPLE

The constant second difference of the quadratic number pattern:
$4 ; x ; 8 ; y ; 20 ; ....$ is 2.

(a) Determine the value of x and y.

(b) Determine which term equals 125.

DO THE SOLUTION

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2.2 The following information is given about a quadratic number pattern:

$T_1 = 3$, $T_2 - T_1 = 9$ and $T_3 - T_2 = 21$

2.2.1 Show that $T_5 = 111$ (2)

2.2.2 Show that the general term of the quadratic pattern is $T_n = 6n^2 - 9n + 6$ (3)

2.2.3 Show that the pattern is increasing for all $n \in N$. (3)

2.2.1	$T_1 = 3$; $T_2 - T_1 = 9$ and $T_3 - T_2 = 21$
	
	3 12 33 66 111
	   9 21 33 45
	     12 12 12
	
	$\therefore T_5 = 3+9+21+33+45 = 111$
	
	OR/*OF*
	$2a = 12$
	$a = 6$
	$3(6)+b = 9$
	$b = -9$
	$6-9+c = 3$
	$T_5 = 6(5)^2 - 9(5)+6 = 111$
		✓ constant second diff = 12
		✓ first differences : 33 and 45
		(2)
		
		OR/*OF*
		✓ constant second diff = 12
		
		✓substitute 5 (2)

2.2.2	$2a = 12$ $a = 6$ $3(6)+b = 9$ or $5\times 6+b = 21$ $b = -9$ $6-9+c = 3$ $c = 6$ $T_n = 6n^2 - 9n + 6$	✓ $2a = 12$ ✓ $3(6)+b = 9 / 5\times 6+b = 21$ ✓ $6-9+c = 3$ (3)
2.2.3	$T_n^{/} = 12n - 9 > 0$ $n > \frac{3}{4}$ $\therefore$ T_n is increasing for $n \in N$ **OR/*OF*** $n = -\frac{b}{2a} = -\frac{-9}{2(6)}$ $n = \frac{3}{4}$ $\therefore$ min at $n = 1$ for $n \in N$ $\therefore$ T_n is increasing for $n \in N$	✓ $T_n^{/} = 12n - 9$ ✓ $n > \frac{3}{4}$ ✓ increasing for $n \in N$ (3) **OR/*OF*** ✓ $n = -\frac{b}{2a} = \frac{9}{2(6)}$ ✓ $n = \frac{3}{4}$ ✓ increasing for $n \in N$ (3)

2. Linear or Arithmetic Sequence: (First difference is a constant)

General Term: $\mathrm{T_n} = a + (n-1)d$

a = first term

n = number of terms

d = common difference = $\mathrm{T_n} - T_{n-1}$

Sum of an Aritmetic Series:

$$\mathrm{S_n} = \frac{n}{2}[2a + (n-1)d]$$

OR

$$\mathrm{S_n} = \frac{n}{2}[a + l]$$

l = last term

EXAMPLE

Consider the following linear number pattern:
2 ; 7 ; 12 ; 17 ; ....

(a) Determine the nth term and hence the 199th term.

(b) Which term of the number pattern is equal to 497?

Solution:

$a)\ a = 2$

$d = 5$

$$\begin{aligned} T_n &= a + (n-1)d \\ &= 2 + (n-1)5 \\ &= 2 + 5n - 5 \\ &= 5n - 3 \end{aligned}$$

$$T_n = 5n - 3$$
$$= 5(199) - 3$$
$$= 992$$

$b)$ $5n - 3 = 497$

$$5n = 500$$
$$n = 100$$

EXAMPLE

$Given: x + 4\ ;\ 2x\ \ x + 8,$

The above is an Arithmetic sequence.

Calculate

The first 3 terms as well as the 12^{th} term

Solution:

$d = T_2 - T_1$
$= (2x) - (x + 4)$
$= x - 4$

$d = T_3 - T_2$
$= (x + 8) - (2x)$
$= 8 - x$

$$x - 4 = 8 - x$$
$$2x = 12$$
$$x = 6$$

$10\ ;\ 12\ ;\ 14$

a = 10 d = 2

$T_n = a + (n-1)d$

$= 10 + (n-1)(2)$

$= 2n + 8$

$T_{12} = 2(12) + 8$

$= 32$

3. Geometric Sequence: (common ratio)

General Term: $T_n = ar^{n-1}$

a = first term

r = common ratio = $\dfrac{T_n}{T_{n-1}}$

n = number of terms

Sum of a Geometric series:

$$S_n = \frac{a(r^n - 1)}{r - 1} \quad ; \quad r > 1$$

$$S_n = \frac{a(1 - r^n)}{1 - r} \quad ; \quad r < 1$$

EXAMPLE

Given: 2 ; 6 ; 18 ; 54

a) Determine the general term.

b) Determine the 10^{th} term.

c) Which term in the sequence will equal 1062882

Solution:

a) $a = 2$

r = 3

$$T_n = ar^{n-1}$$

$$T_n = 2(3)^{n-1}$$

$$b) \quad T_{10} = 2(3)^9$$

$$= 39366$$

$$c) \quad 2(3)^{n-1} = 1062882$$

$$(3)^{n-1} = 531441$$

$$(3)^{n-1} = (3)^{12}$$

$$n - 1 = 12$$

$$n = 13$$

EXAMPLE

$7x + 1$; $2x + 2$; $x - 1$ is a Geometric Sequence

a) Determine x

b) Determine the first 3 terms

Solution:

$a) \quad r = \dfrac{2x+2}{7x+1} \quad ; \quad r = \dfrac{x-1}{2x+2}$

$$\frac{2x+2}{7x+1} = \frac{x-1}{2x+2}$$

$(2x+2)(2x+2) = (x-1)(7x+1)$

$4x^2 + 8x + 4 = 7x^2 - 6x - 1$

$3x^2 - 14x - 5 = 0$

$(3x+1)(x-5) = 0$

$x = \dfrac{-1}{3} \ or \quad x = 5$

$7x+1\ ;\ 2x+2 \quad x-1$

Sequence 1 :

$\dfrac{-4}{3};\dfrac{4}{3};\dfrac{-4}{3}$

Sequence 2 :

36 ;12; 4

<u>EXAMPLE</u>

3 ; *a* ; *b* are the first three terms of an Arithmetic sequence. If the third term is increased by 3, the three terms form a geometric sequence.
Calculate the values of *a* and *b.*

DO THIS SUM ON YOUR OWN